

Growth States and Shocks

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Abstract

GDP growth typically vibrates with modest variation around a mean of a few percent per year, but periodically, mean growth undergoes a major shift, vibrating thereafter around a new level. I present a transmission mechanism with nonlinear dynamics that endogenously translates random sectoral shocks into just this sort of behavior, creating what might be thought of as multiple growth states. Small shocks cause vibration within a state. Sufficiently large shocks cause a state-change. Behind the nonlinear dynamics lies essentially the same dynamic externality that drives the 'new growth theory' models. The relative balance of endogenous versus exogenous growth determines whether the economy will have multiple stable growth states. The model can generate data which looks much like the data generated by a Markov process of the sort identified in Hamilton (1989) or a trend-breaking process as in Perron (1989). The model has two output processes. Input factors are drawn into the 'leading process,' where learning-by-doing further increases that process's technological lead. If the 'leading process' is also the inherently high-growth process, then growth is fast for both technologies. Shocks to preferences and technologies cause endogenous switching of the leading sector role between the high-growth and slow-growth processes.

Keywords: multiple equilibria, endogenous growth, Markov-switching

JEL classification: E32, O30, O41

1. Introduction

Economic growth is not smooth. GDP growth typically vibrates with modest variation around a mean of a few percent per year, but periodically, mean growth undergoes a major shift, vibrating thereafter around a new level. So states a stylized version of the findings of the state-switching models of Hamilton (1989) and the trend-breaking models of Perron (1989). In this paper, I present a transmission mechanism with nonlinear dynamics that endogenously translates random shocks into just this sort of behavior. The transmission mechanism creates multiple growth states. Small shocks cause vibration within a state. Sufficiently large shocks cause a state-change. Behind the nonlinear dynamics lies the same dynamic externality that drives the 'new growth theory' models of Romer (1990) and Lucas (1988). Thus the economic principles explaining very long-run growth can also contribute to our understanding of higher frequency fluctuations.

The model economy has two production processes whose outputs are good, although not necessarily perfect substitutes. Each output is produced using a process-specific technology and a share of the single input factor, which is in fixed supply. Productivity increases over time in two ways. First, there is Solovian exogenous technological change. Each technology has an own-growth rate but also benefits from cross-fertilization from the other process. Second, technology improves through endogenous technological change. Because of learning-by-doing, or any of the myriad other reasons developed in the new growth literature, the greater the intensity of use of the productive input in a process, the faster the growth of its process-specific technology. The exogenous growth component, specifically the cross-fertilization aspect, tends to stabilize the economy. The endogenous component provides, as usual, a source of positive feedback which tends to destabilize the economy.

Characterization of the economy with multiple growth states, two for example, is relatively straightforward. Each of the growth states has an 'attraction basin' within which the economy's endogenous mechanics move growth toward a local steady-state. Small shocks

cause vibration within an attraction basin. A large shock, in the right direction, shifts the economy into the attraction basin of the other state. In addition to these general characteristics, a model with multiple growth states can generate data that looks much like the data generated by a Markov process of the sort identified in Hamilton (1989). Generally, the model yields a nonlinear differential equation for growth that might be well approximated by a Markov process even though it does not have an exact finite-state Markov representation. I look at a special case that is very close to a two-state Markov process, specifically in that the growth rate changes discontinuously across a state transition. This model has the additional advantage of being analytically tractable.

A formal presentation of the model appears below. The central intuition is that the presence of a dynamic externality tears apart the contemporaneous and intertemporal consequences of allocation decisions made by individual agents. The specific model implemented here is rather simple. Call the two processes X and Y . Suppose Y can be thought of as the 'high growth' process, either because technological change is inherently faster or because the opportunities for learning-by-doing are greater. At a given instant, either the Y or X has relatively higher productivity and can be thought of as the 'leading process.' Mean growth fluctuates as the high and low growth processes (defined by the time derivatives of technology) jockey for position as the leading process (defined by the levels of technology).

The key state variable in the model is the relative price of one output in terms of the other. If process Y is unusually productive, then its output will be high and its relative price will be low; the low price stimulates a large volume of demand. Most of the input factor will move into Y , so long as demand is sufficiently elastic to offset the low inherent factor demand due to high productivity. The endogenous growth externality will cause productivity to rise relatively faster for Y , pulling even more resources into it. This is the fundamental source of positive feedback in the model. Because both technologies are used in the production of new technologies for both processes, the high productivity process pulls growth in the entire economy along after it. If the externality plays a sufficiently important role in technological

growth, then there will be two stable steady-state equilibria. In one, the high growth process dominates and the economy grows quickly. In the other, the low growth process is the technological leader and the economy grows slowly.

2. Related Literature

There is both a theoretical and empirical literature which bears on the topic at hand.

The two closest theoretical articles are Durlauf (1993) and Lucas (1988). Durlauf (1993) introduces the use of random field theory to macroeconomics, presenting a model of multi-sector nonergodic growth. (See also Durlauf (1991) and Quah (1990).) The structure of Durlauf's model is rather different from the structure here; in particular 'cross-fertilization' is a source of positive feedback, as opposed to the stabilizing role played here. In Durlauf's model each process needs technological improvement from the other. There can be a low-level trap in which neither process takes off because the other process hasn't. Multiple equilibria arise from the possibility that either both processes or neither process grow quickly. Here, in contrast, multiple equilibria arise when either an inherently high-growth or low-growth process can become the engine of growth.

The microeconomic underpinnings of the model are closest to Lucas (1988). The model may be thought of, in part, as an implementation of the theory of endogenous comparative advantage. This aspect of the economics is closely related to Krugman's (1987) "Narrow Moving Band," and Lucas' (1988, section 5) "Learning-by-doing and comparative advantage." However, the two possible outcomes in the Lucas model are either a single stable steady or eventual specialization in one process. The model here allows for multiple equilibria using both processes, which is necessary if there is to be potential switching back-and-forth between the equilibria.

Acemoglu and Scott (1997) present a model of high frequency state-switching¹ which relies on intertemporal increasing returns to scale which are internal to the firm. Fixed costs lead to zero/one investment decisions on the part of each firm. When shocks are highly

correlated across firms, so are investment decisions. In the limit, perfectly correlated investment decisions are realized as Hamilton style regime switching where persistence depends on the propagation of the shocks through the intertemporal increasing returns. Acemoglu and Scott invoke fixed costs and internal increasing returns to generate regime switching; the model presented here doesn't have fixed costs, but requires external increasing returns.

Cooper (1994) presents a model in which shocks can generate serially correlated state shifts between a high fixed-cost/high marginal productivity and a low fixed-cost/low marginal productivity technology. At low levels of aggregate demand all firms choose the low level technology and analogously at high levels. For intermediate levels of aggregate demand, multiple equilibria are possible and, by assumption, firms choose the technology in use at the last unique equilibria. Moderate changes in aggregate demand move the economy into the region in which history matters, thereby generating state persistence very much like the kind seen below. Cooper's mechanism is much more Keynesian than the one proposed here, as persistence arises out of an assumed persistence in the coordination mechanism, rather than the theory of endogenous comparative advantage used here.

The original empirical piece is Lilien (1982). Lilien demonstrates the importance of sectoral shifts for understanding aggregate economic activity, although the theoretical model given is unrelated to the one here. Hamilton (1989) shows that the economy may be modeled as a Markov process rather than a linear ARIMA process and provides evidence that such a process can match business cycles. (For a recent survey on Hamilton style models, see Diebold and Rudebusch (1994).) Perron (1989) suggests the economy is well-approximated by a trend-stationary process with low frequency, every few decades, changes in trend.

More generally the theoretical genesis of the model here lies in the 'new growth theory' of Romer (1986, 1990), Lucas (1988), and others.² Several authors have multi-sector new growth models. Mulligan and Sala-i-Martin (1993) examine the behavior of two-sector endogenous growth models. In the last section of the paper I turn to an examination of

sectoral shifts and long-run growth, so it should be pointed out that there are other mechanisms which can generate multiple growth equilibria. Galor (1996) points out several ways in which multiple equilibria (club convergence in this case) can arise in neoclassical models. In particular, a linkage between saving and the wage share can be sufficient, as can unequal distributions of income in the presence of capital market imperfections, as can endogenous fertility. Azariadis (1996) also reviews a number of possible sources of poverty traps, including technology traps. Galor and Tsiddon (1997) present a model in which reallocation of an input factor between sectors, high-ability/high-education workers in their case, endogenously controls technological progress. This reallocation plays a role similar to the one in this paper.

3. The Model

Competitive firms produce consumption goods X and Y using a single input factor H which is in fixed supply in the economy but completely mobile between firms.³ The production function for Y is $Y = A_Y b_Y H_Y$; the function for the X sector is symmetric. A_Y represents technology which improves over time, b_Y is a constant-over-time productivity factor, and H_Y is the amount of the input factor used in the Y process. Given the simple structure of production, p , the relative price of X in terms of Y is given directly by the ratio of marginal costs, $p = A_Y b_Y / A_X b_X$.

The demand side of the economy consists of a large number of identical consumers for whom X and Y are relatively good, perhaps perfect, substitutes. Preferences of the representative consumer are given by $U(Y, X) = c_Y Y^r + c_X X^r$, $c_Y, c_X > 0, 0 < r \leq 1$. Consumers equate the price p to the marginal rate of substitution, $p = c_X / c_Y (X/Y)^{r-1}$.

Together, preferences and current technology determine how society allocates $H = H_Y + H_X$ between the two processes. It turns out to be useful to express factor allocations as a function of the price.

$$H_Y = \frac{(kp)^{\frac{r}{1-r}}}{1 + (kp)^{\frac{r}{1-r}}} H, \mathbf{k} \equiv \left(\frac{c_Y}{c_X} \right)^{\frac{1}{r}}, H'_Y \equiv \frac{dH_Y}{dp} = \frac{\mathbf{r}}{1-\mathbf{r}} \frac{1}{p} \frac{H_Y}{H} [H - H_Y] \quad (1)$$

Technological advances for each process depend on the existing level of technology for both goods and, either because of learning-by-doing or some of the other reasons developed in the new growth literature, on the intensity of use of the productive input in that sector. As is usual in this literature, I assume that the role of the productive input in technological change is purely external to the individual firm. Here, this assumption buys us two results. In the first place, it makes it easier for the model to produce multiple steady-state equilibria. Since the model is in other ways perfectly competitive and has identical agents, if technological change were completely internal, then there would be a unique first-best path. However, even a small external role will permit multiple steady-states. The second 'result' is that making technological change completely, rather than partly, external eliminates the need for each firm to solve an analytically intractable dynamic programming problem.⁴

Specifically, I assume the dynamics of technological change are as given in (2).

$$\dot{A}_Y = \mathbf{a}_Y A_Y + \mathbf{g}_Y A_X + \mathbf{d}_Y A_Y H_Y \text{ and } \dot{A}_X = \mathbf{a}_X A_X + \mathbf{g}_X A_Y + \mathbf{d}_X A_X H_X, \mathbf{a}, \mathbf{g}, \mathbf{d} > 0 \quad (2)$$

Note first that in the absence of endogenous technological change, i.e. $\mathbf{d} = 0$, equation (2) describes a growth model that happens to have two goods. If $\mathbf{d} = 0$ there is a unique, interior, stable steady-state in which both sectors grow at the same rate with a constant relative price, so that the two goods may be treated as a composite commodity. In other words, the feedback between technologies is, by construction, stabilizing and leads to only a single steady-state. Multiple steady-states are due to endogenous technological change flowing from the allocation of the input factor.

The key to understanding the inter-sectoral allocation of H , and from this the dynamics of growth, is to track the relative price p . Defining $\dot{a}_Y \equiv \dot{A}_Y / A_Y$ and $\dot{a}_X \equiv \dot{A}_X / A_X$, (2) can be rewritten as

$$\dot{a}_Y = a_Y + g_Y \cdot \left(\frac{b_Y}{b_X}\right) \cdot p^{-1} + d_Y H_Y \text{ and } \dot{a}_X = a_X + g_X \cdot \left(\frac{b_Y}{b_X}\right)^{-1} \cdot p + d_X H_X \quad (3)$$

Taking logs of the price equals marginal cost condition and differentiating with respect to time gives us (4).

$$\frac{\dot{p}}{p} = \dot{a}_Y - \dot{a}_X = \left\{ a_Y + g_Y \cdot \left(\frac{b_Y}{b_X}\right) \cdot p^{-1} + d_Y H_Y \right\} - \left\{ a_X + g_X \cdot \left(\frac{b_Y}{b_X}\right)^{-1} \cdot p + d_X H_X \right\} \quad (4)$$

Equation (1) and equation (4) give a nonlinear differential equation for p which together with initial conditions completely describes the dynamics of the economy. Output growth reflects technology growth and, out of the steady-state, reallocation of the input factor.

If s is Y 's share in GDP we get the natural formulation for real GDP growth

$$\frac{\dot{GDP}}{GDP} = \frac{\dot{Y}}{Y} s + \frac{\dot{X}}{X} (1 - s).$$

The combination of cross-fertilization and endogenous reallocation of the input factor is sufficient to generate interesting transitional dynamics. Figure 1 shows an example where growth rates are nonmonotonic over time.⁵ Suppose the Y process has a faster exogenous growth component, but that the X process starts with a higher level of technology. Initially, most production is by X , with low but gradually increasing growth. Eventually, Y technology improves to the point that a significant fraction of the input factor moves into the Y process. This reallocation causes a transitional drop in the growth rate, as fast growth in Y applies to a relatively small share of the level of production. The growth slow down is followed by a growth leap, as fast reallocation of the input factor leads Y to overtake X as the primary means of production. (The key relation is equation (1) which gives H'_Y as a logistic function of H_Y .) Thus the adjustment to steady-state growth can show slowly increasing growth, a slow down, a sudden spurt, and then a gradual approach to the steady-state.

[Figure 1 goes about here]

In general, the easy way to analyze this system is to plot the two sectoral growth rates, \dot{a}_Y and \dot{a}_X , as functions of the relative price. The vertical distance between the two functions gives \dot{p}/p . Figure 2 shows the growth rates of technology absent endogenous technological

change, $\delta=0$. Here the intensity of input factor use plays no role. There is a unique, stable, steady-state at p^* . When we allow $d > 0$, as we do from this point, exogenous technological growth remains a stabilizing force, even when endogenous technological change adds a destabilizing and sometimes dominating element. The existence of stable steady-states is summarized in the following proposition.

[Figure 2 goes about here]

Proposition: The system always has a stable internal equilibrium and the leftmost and rightmost equilibria are always stable.

Proof: Note $p \in (0, \infty)$ and that H_Y, H_X are bounded. Multiply equation (4) by p and note as $p \rightarrow 0$, $\dot{p} \rightarrow g_Y \cdot \left(\frac{b_Y}{b_X}\right) > 0$. As $p \rightarrow \infty$, $\frac{\dot{p}}{p^2} \rightarrow -g_X \cdot \left(\frac{b_Y}{b_X}\right)^{-1} < 0$.

The economy has multiple stable steady-states when there is an intervening unstable steady-state, that is if the \dot{a}_Y and \dot{a}_X lines cross with \dot{a}_Y striking from below, as in Figure 3. In general, the condition for multiple stable steady-states is for there to be a steady-state value where $p \cdot (d_Y + d_X)H'_Y > p \cdot g_X \cdot \left(\frac{b_Y}{b_X}\right)^{-1} + g_Y \left(\frac{b_Y}{b_X}\right) \cdot p^{-1}$.

Consider first the symmetric case, where parameters are equal across sectors. There exists a steady-state (obviously) at $p^* = 1$, which is unstable steady-state iff

$dH'_Y = d \frac{r}{1-r} \frac{H}{4} > g$. We can see that four elements determine whether there are multiple steady-states. A large marginal importance of the externality, high δ , or a small cross-component in technological growth, low γ , cause multiple steady-states. Greater elasticity of substitution, causing greater positive feedback from technology to factor demands, leads to multiple steady-states. Finally, whatever the parameters, a sufficiently large H generates multiple steady-states.⁶

The symmetric case is analytically tractable as some of the nonlinearity disappears, but not otherwise all that interesting because the two stable steady-states have the same growth rate. More generally, the growth rates for the two technologies are as illustrated in Figure 3. The heights of the intersecting curves at the two stable roots give mean GDP growth

in the respective states. Mean growth is determined by both the exogenous (\mathbf{a} and \mathbf{g}) and endogenous (\mathbf{d} and H) sources of technological change. In particular, equal changes in \mathbf{a}_Y and \mathbf{a}_X scale growth without changing the steady-states.

[Figure 3 goes about here]

4. Stochastic shocks

The interesting shocks⁷ in this model are differential shocks to Y and X that have some finite mass over time. Any shock, or sequence of shocks, which causes a state switch generates a change in output and technology growth which endures until a state-reversing shock comes along. Since the accumulated technology remains intact through future state-switches, a short-lived shock to either technology or preferences causes a permanent change to the level of output.

Consider two simple kinds of stochastic shocks. The first is a shock to technology of the real business cycle sort. A random discovery, that is one in excess of the movement of technology described by equation (3), pushes A_Y up relative to A_X . The relative price rises and the economy as shown in the Figures moves to the right. The positions of the \dot{a}_Y and \dot{a}_X curves remain unchanged. A small shock to relative technology leaves you in the attraction basin for the currently leading technology. A larger shock in the appropriate direction knocks you into the other one. If the price is near the unstable steady-states, then a very small shock will suffice.

The second kind of shock is a preference shift, modeled as a shift in c_Y versus c_X . Note that demand shifts move κ but not the current price. So a demand shift changes the location of the two growth curves in the Figures but not the current position on the graph. The effect when $\mathbf{r} \approx 1$, shown in Figure 4 below, is particularly interesting. The stable steady states are independent of κ , but the unstable point is at exactly $1/\kappa$. Here, the effect of a relative demand shift is to move the boundary defining the two attraction basins. A demand shock which is small enough, relative to the economy's current position, that no state switch results has no

real effect, since reallocation of the input factor is negligible. In contrast, a demand shift large enough to move $\frac{1}{k}$ past the current price and persistent enough to allow time for the price to move endogenously past the old $\frac{1}{k}$ point causes a permanent change in the level of technology and output. Thus, small relative demand shocks have no effect whatsoever on output while larger ones have permanent effects.

5. A Limiting Model

In this section I consider the limiting behavior of the system as ρ approaches one. Doing so is useful both because analytic solutions are available for a number of interesting characteristics and because the limiting model gives the closest approximation to the Markov process of Hamilton (1989). Figure 4 shows the growth rates of the two technologies at the limiting case, $\mathbf{r} = 1$.

[Figure 4 goes about here]

In equation (4) the exponent $\frac{r}{1-r}$ goes to infinity as $\mathbf{r} \rightarrow 1$.⁸ For $p > \frac{1}{k} = \frac{c_x}{c_y}$, H is allocated entirely to good Y and vice versa for $p < \frac{1}{k}$. The growth rate of Y is a broken hyperbola, shifted vertically $\mathbf{d}_y H$ at $p = \frac{1}{k}$. Similarly, the growth rate of X is piece-wise linear, shifted vertically $-\mathbf{d}_x H$ at $p = \frac{1}{k}$.

There are two stable and one unstable steady-states iff the growth lines in Figure 4 cross at $p = \frac{1}{k}$ – large H is sufficient to guarantee multiple steady-states. The left and right stable-steady states are, respectively,

$$\begin{aligned} p_L^* &= \frac{1}{2\mathbf{g}_x} \left(\frac{b_y}{b_x} \right) \left[(\mathbf{a}_y - \mathbf{a}_x - \mathbf{d}_x H) + \sqrt{(\mathbf{a}_y - \mathbf{a}_x - \mathbf{d}_x H)^2 + 4\mathbf{g}_x \mathbf{g}_y} \right] \\ p_R^* &= \frac{1}{2\mathbf{g}_x} \left(\frac{b_y}{b_x} \right) \left[(\mathbf{a}_y - \mathbf{a}_x + \mathbf{d}_y H) + \sqrt{(\mathbf{a}_y - \mathbf{a}_x + \mathbf{d}_y H)^2 + 4\mathbf{g}_x \mathbf{g}_y} \right] \end{aligned} \quad (5)$$

Equations (5) show that the two roots are pushed apart by large $\mathbf{d}H$ and small γ . For example, under symmetry, $p_R^* - p_L^* = \mathbf{d}H/\mathbf{g}$. Asymmetry depends on the centering of the unstable root in comparison to the stable roots. The unstable root is at $\mathbf{k}^{-1} = c_x/c_y$ and so

depends only on tastes, while the other factors determine the stable positions. Differential tastes for the two goods are therefore a likely contributor to asymmetry.

The limiting model provides one possible theoretical underpinning for the kind of regime-switching models which have become popular following Hamilton (1989). The model is ‘approximately Markov’ in the sense that the two possible values for H_y define two states within which output growth follows a mean reverting process. As can be seen in Figure 4, the growth rate takes a discrete jump when transiting between states. However, given a constant variance shock process, the transition probabilities depend on how close p is to a basin boundary so the overall process does not exactly obey a Markov rule.

An illustrative simulation roughly matches Hamilton’s estimates and points out a difference in terms of selection bias between exogenous (Hamilton) and endogenous (this model) switching. An artificial 20,000 quarter sample was generated by adding i.i.d. proportional shocks to A_y/A_x .⁹ The high steady-state growth rate was 1.2 and the low rate was 0.4. Note that the latter is higher than Hamilton’s estimate, since the model here doesn’t sensibly permit negative steady-state growth. However, it is noteworthy that observed mean growth in the low growth state was below the steady-state rate, 0.2 rather than 0.4. With endogenous state-switching, there is some selection bias in that the economy is more likely to be in a low growth state following a negative shock. The estimated switching probabilities in the simulation were .99 and .85, both somewhat higher than Hamilton’s estimates. Estimation of an AR(4) process showed large positive coefficients at lags 1 and 3.

5. Implications for Growth

Much of the economics driving the model is borrowed from new growth theory, so it seems worthwhile to ask whether the model can make a return contribution to the study of growth. (To be clear, a single set of parameters and a single interpretation of the model is *not* going to explain *both* high and low frequency changes. The interpretation in this section emphasizes shifts between sectors and low frequency trend breaks.) The model here provides

an alternative, or perhaps complementary, explanation for “nonconvergence.” Further, there is some mild empirical evidence that suggests that sectoral realignment is an important key to growth.

Much of the empirical growth literature centers on measuring the average speed of convergence of countries or regions. However, there is a growing set of evidence on the cross-sectional distribution of growth arguing in favor of instances of divergence. Durlauf and Johnson (1995) and Quah (1995, 1996) provide formal econometric evidence for multiple steady-states. Quah (1995) says “The picture that emerges is one of a world where countries tend – in the long run – towards either the very rich or the very poor, with the middle income classes disappearing. The disparity between the rich and the poor, further, appears to be widening.” Quah (1996) states “The data show little cross-country convergence; instead, the important features are persistence, immobility, and polarization, exemplified by ‘convergence club’ or ‘twin peaks’ dynamics. Durlauf and Johnson write “... the marginal product of capital is shown to vary with the level of economic development. These results are consistent with growth models which exhibit multiple steady states. Multiple growth steady-states can, of course, arise for a variety of reasons other than the one proposed here. Azariadis and Drazen (1990) provide an early and important example. Acemoglu and Zilibotti (1997) demonstrate that the randomness of “take-off” may result from limits on risk-pooling. Matsuyama (1991) derives multiple steady-states through the assumption of increasing returns to manufacturing as opposed to agriculture.

It is worthwhile to provide some basic empirical evidence in favor of the importance of sectoral shifts. In particular, since H is constant in the model here, we would like evidence of a sectoral shift that matters after controlling for the aggregate level of human capital. The obvious candidate sector is agriculture. The evidence of a shift out of agriculture being associated with increased growth is strong enough to be taken at least as a clear *prima facie* case. Young’s (1995) examination of growth in the East Asian tigers emphasizes the importance of the intersectoral transfer of labor out of agriculture into manufacturing,

particularly for Taiwan and Korea. So the outstanding recent example of countries switching from the low growth to the high growth club includes a strong sectoral shift. Barro and Sala-i-Martin's (1992) cross state regressions also show the importance of a shift out of agriculture. To control for idiosyncratic shock effects on state income growth, Barro and Sala-i-Martin construct a sectoral composition variable. In general, the effect of this variable is positive. However, due to data limitations the pre-1930 measurement of "sectoral composition" was simply fraction of national income originating in agriculture. This effect of *this* variable on growth is significantly negative. (Barro and Lee, Table 1, lines 12-14.)

6. Summary

Endogenous-technological-change-external-to-the-firm leads to an economy with multiple stable growth states. Specifically, in this model a relatively large endogenous component to technological change generates two stable growth states while a large exogenous component leads to a single steady-state. Relative shocks to either technology or demand can move the economy across growth states. Temporary shocks can have permanent effects. Large shocks can have disproportionately larger effects than small shocks. When thinking of relatively long horizons, it may be better to think of 'shifts' rather than 'shocks,' as there is nothing in the model requiring changes to be unanticipated. Indeed, the model here can be used to explain growth traps and even 'immiserising growth,' since a technological improvement in the low-growth sector can increase output but lower the long run growth rate.

The new growth literature suggests that technological change which is endogenous to the economy but external to firms helps explain long horizon growth. The model here suggests that the same forces contribute to understanding nonlinear dynamic fluctuations at shorter horizons as well.

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¹ See also Durlauf (1991b), and Hamilton (1988).

² See Barro and Sala-i-Martin (1995). As an interesting aside, Leontief (1958) presented a model with stable low-level and high-level equilibria for GNP as early as 1958.

³ The single input factor assumption buys considerable simplification at the cost of making the link between factor allocation and induced growth somewhat unrealistic. More realistically, suppose there are relatively few research workers and relatively many production workers and that only the presence of the former leads to induced growth. So long as research workers switch easily to the 'hot' technology state switching can occur even though total labor is largely immobile.

⁴ Because the role of the input factor in technological change is external to the firm and because the input factor is completely mobile, the firm values the input only for its contribution to current production. In contrast, the technical difficulty of the completely general formulation has forced much of the literature to focus attention solely on stable-steady states, a situation which may be harmless for studying very long run growth, but which is discomfiting at higher frequencies. To quote Mulligan and Sala-i-Martin (1993) in regard to the more general case, "The transitional dynamics of endogenous growth are not well understood."

⁵ The simulation in Figure 1 uses parameters $\mathbf{a}_y = .04$, $\mathbf{a}_x = .02$, $\mathbf{g} = \mathbf{d} = .01$, $b = c = 1$, $\mathbf{r} = .9$, $H = 2$, and initial conditions $A_y^0 = .1$, $A_x^0 = 1$.

⁶ The model is sufficiently developed at this point to draw comparisons between it and the model in section 5 of Lucas (1988). The microeconomic assumptions leading to endogenous growth are identical.

The differences arise out of the endogenous movement of H across sectors, specifically the Lucas model has $\mathbf{a}_y = \mathbf{a}_x = \mathbf{g}_y = \mathbf{g}_x = 0$. The two possible outcomes in the Lucas model are either a single stable steady state, if the two goods are poor substitutes, or complete specialization in one good. In the latter case, the comparative advantage of the produced good grows infinitely large. The kind of two-sector multiple equilibrium results discussed here aren't possible in the Lucas model. (Nor were they the goal of that model.)

⁷ Since the dynamics of technological change are external to the firm *and* there is no capital accumulation, it makes no difference whether 'shocks' are anticipated or unanticipated.

⁸ The limiting arguments behave perfectly well as $\rho \rightarrow 1$, although if $\rho = 1$ one should probably identify p as the 'supply price.'

⁹ Simulation parameters were

$[\mathbf{a}_y \ \mathbf{a}_x \ \mathbf{g}_y \ \mathbf{g}_x \ \mathbf{d}_y \ \mathbf{d}_x \ b_y \ b_x \ c_y \ c_x \ \mathbf{s}]$
 $= .052[.001 \ .001 \ .07 \ .05 \ .0025 \ .0005 \ 1 \ 1 \ 1 \ 1 \ 1.5]$ and $H = 85$. The random shocks \mathbf{e} were generated by $\mathbf{e} = e^u / \text{mean}(e^u)$ where $u \sim N(0, \mathbf{S}^2)$.

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Captions

Figure 1: Example path of nonmonotonic adjustment to the steady-state.

Figure 2: Equilibrium growth with unique steady-state.

Figure 3: Equilibrium growth with multiple steady-states.

Figure 4: Equilibrium growth in a Markov model.

Figures

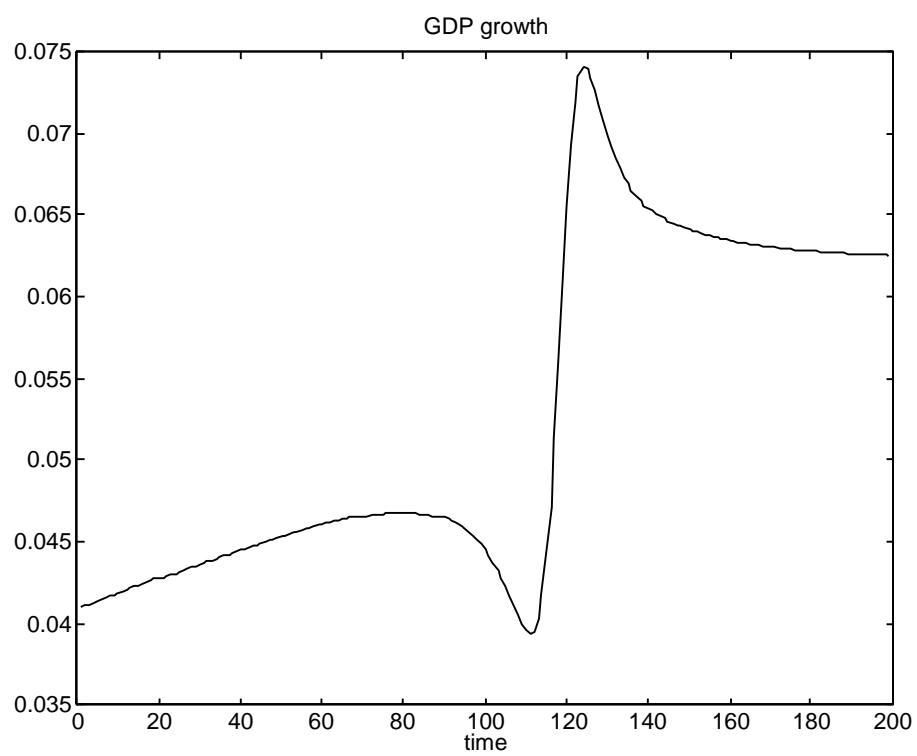


Figure 1

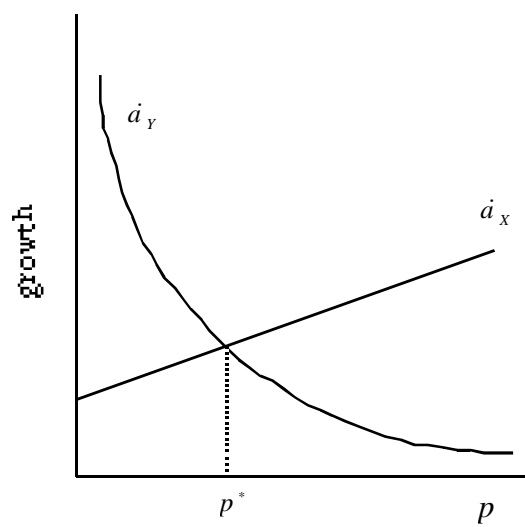


Figure 2

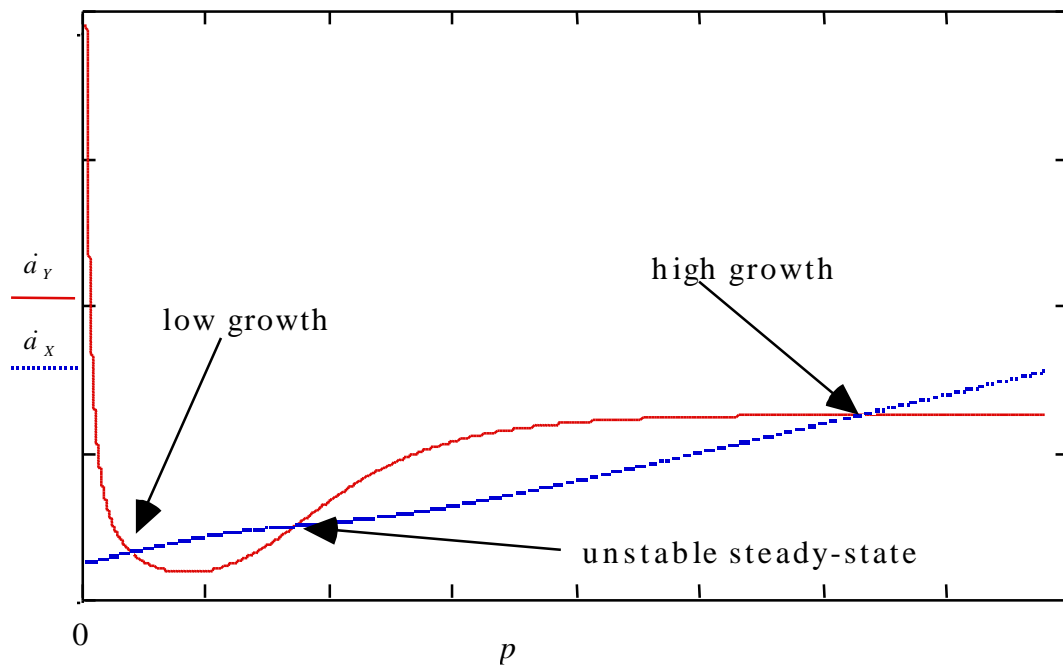


Figure 3

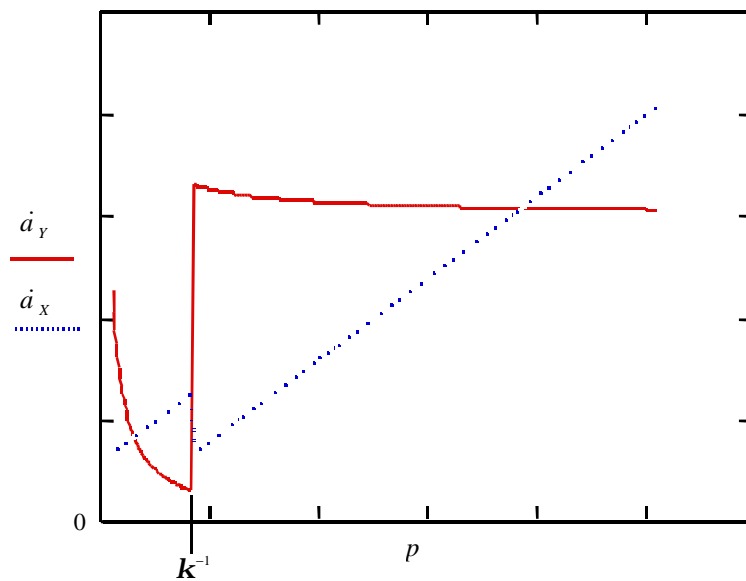


Figure 4

Appendix - Not for publication

Derivations are presented here in a fair amount of detail.

The production function for the Y process is $Y = A_Y b_Y H_Y$

p , the relative price of X in terms of Y is given directly by the ratio of marginal costs.

$$p = \frac{A_Y b_Y}{A_X b_X} = \frac{MC_X}{MC_Y} \quad (A1)$$

Preferences of the representative consumer are given by

$$U(Y, X) = c_Y Y^r + c_X X^r, \quad c_Y \cdot r > 0, \quad c_X \cdot r > 0, \quad r < 1 \quad (A2)$$

Consumers equate the price p to the marginal rate of substitution.

$$p = \frac{c_X}{c_Y} \left(\frac{X}{Y} \right)^{r-1} = \frac{MU_X}{MU_Y} \quad (A3)$$

Substituting the production functions in for X and Y we get

$$p = \frac{A_Y b_Y}{A_X b_X} = \frac{c_X}{c_Y} \left(\frac{X}{Y} \right)^{r-1} = \frac{c_X}{c_Y} \left(\frac{A_X b_X H_X}{A_Y b_Y H_Y} \right)^{r-1}$$

$$p = \frac{c_X}{c_Y} \left(\frac{H - H_Y}{p H_Y} \right)^{r-1} \quad \text{or} \quad p^r \cdot \left(\left(\frac{c_Y}{c_X} \right)^{1/r} \right)^r = \left(\frac{H}{H - H_Y} \right)^{1-r}$$

$$H_Y = \frac{(k\phi)^{1/r}}{1 + (k\phi)^{1/r}} H, \quad k \equiv \left(\frac{c_Y}{c_X} \right)^{1/r} \quad (A4)$$

The derivative of H_Y with respect to p is

$$H_Y' \equiv \frac{dH_Y}{dp} = \frac{r}{1-r} \frac{1}{p} \frac{H_Y}{H} [H - H_Y] \quad (A5)$$

and we assume $r > 0$.

The dynamics of technological change are as given in (A6).

$$\dot{A}_Y = a_Y A_Y + g_Y A_X + d_Y A_Y H_Y \quad \text{and} \quad \dot{A}_X = a_X A_X + g_X A_Y + d_X A_X H_X, \quad a, g, d > 0 \quad (A6)$$

The key to understanding the inter-sectoral allocation of H , and from this the dynamics of growth, is to track the relative price p . Substituting in from (A1) for A_x/A_y and defining $\dot{a}_y \equiv \dot{A}_y/A_y$ and $\dot{a}_x \equiv \dot{A}_x/A_x$, (A6) can be rewritten as

$$\dot{a}_y = \mathbf{a}_y + \mathbf{g}_y \cdot \left(\frac{b_y}{b_x}\right) \cdot p^{-1} + \mathbf{d}_y H_y \text{ and } \dot{a}_x = \mathbf{a}_x + \mathbf{g}_x \cdot \left(\frac{b_y}{b_x}\right)^{-1} \cdot p + \mathbf{d}_x H_x \quad (\text{A7})$$

Taking logs of prices from (A1), differentiating with respect to time, and using (A7) gives us (A8).

$$\frac{\dot{p}}{p} = \dot{a}_y - \dot{a}_x = \left\{ \mathbf{a}_y + \mathbf{g}_y \cdot \left(\frac{b_y}{b_x}\right) \cdot p^{-1} + \mathbf{d}_y H_y \right\} - \left\{ \mathbf{a}_x + \mathbf{g}_x \cdot \left(\frac{b_y}{b_x}\right)^{-1} \cdot p + \mathbf{d}_x H_x \right\} \quad (\text{A8})$$

Inserting equation (A4) into equation (A8) give a nonlinear differential equation for p .

Output growth reflects technology growth and, out of the steady-state, reallocation of the input factor.

$$\frac{\dot{Y}}{Y} = \dot{a}_y + \frac{H'_y}{H_y} \cdot \dot{p}$$

If we define the output price index $q \equiv p_Y^s p_X^{1-s}$, where p_Y , p_X and q are nominal prices and s is Y 's share in GDP, and define real GDP as $GDP = \frac{p_Y}{q} \cdot Y + \frac{p_X}{q} \cdot X$, then we get the natural formulation for real GDP growth

$$\frac{\dot{GDP}}{GDP} = \frac{\dot{Y}}{Y} s + \frac{\dot{X}}{X} (1-s).$$

Proposition: The system always has a stable internal equilibrium and the leftmost and rightmost equilibria are always stable.

Proof: Note $p \in (0, \infty)$ and that H_y, H_x are bounded. Multiply equation (A8) by p and note as $p \rightarrow 0$, $\dot{p} \rightarrow \mathbf{g}_y \cdot \left(\frac{b_y}{b_x}\right) > 0$. As $p \rightarrow \infty$, $\frac{\dot{p}}{p} \rightarrow -\mathbf{g}_x \cdot \left(\frac{b_y}{b_x}\right)^{-1} < 0$.

There are multiple steady-states if there is a value of p such $\dot{a}_y(p) = \dot{a}_x(p)$ and $\dot{a}_y(p)' > \dot{a}_x(p)'$ – graphically, that the Y growth line is steeper than the X growth line. Note that

$$\dot{a}_Y(p)' = -\mathbf{g}_Y\left(\frac{b_Y}{b_X}\right)p^{-2} + \mathbf{d}_Y H_Y' \text{ and } \dot{a}_X(p)' = \mathbf{g}_X\left(\frac{b_Y}{b_X}\right)^{-1} - \mathbf{d}_X H_Y' \quad (\text{A9})$$

In general, the condition for multiple stable steady-states is for there to be a steady-state value where

$$p \cdot (\mathbf{d}_Y + \mathbf{d}_X) H_Y' > p \cdot \mathbf{g}_X \cdot \left(\frac{b_Y}{b_X}\right)^{-1} + \mathbf{g}_Y\left(\frac{b_Y}{b_X}\right) \cdot p^{-1}. \quad (\text{A10})$$

A Limiting Model

In this section I consider the limiting behavior of the system as ρ approaches one.

Therefore, the only thing required for there to be multiple steady-states is for the neighborhood around $p \approx \frac{1}{k}$ to include a steady-state. From (A7), there exists a steady-state in that neighborhood iff

$$\begin{aligned} \exists H_Y \in (0, H), \text{ s.t. } \Omega + (\mathbf{d}_Y + \mathbf{d}_X) H_Y - \mathbf{d}_X H &= 0 \\ \Omega \equiv (\mathbf{a}_Y - \mathbf{a}_X) + \left(\mathbf{g}_Y\left(\frac{b_Y}{b_X}\right) \cdot \mathbf{k} - \mathbf{g}_X\left(\frac{b_Y}{b_X}\right)^{-1} / \mathbf{k} \right) \end{aligned} \quad (\text{A11})$$

The left and right stable-steady states are, respectively,

$$\begin{aligned} p_L^* &= \frac{1}{2\mathbf{g}_X\left(\frac{b_Y}{b_X}\right)} \left[(\mathbf{a}_Y - \mathbf{a}_X - \mathbf{d}_X H) + \sqrt{(\mathbf{a}_Y - \mathbf{a}_X - \mathbf{d}_X H)^2 + 4\mathbf{g}_X\mathbf{g}_Y} \right] \\ p_R^* &= \frac{1}{2\mathbf{g}_X\left(\frac{b_Y}{b_X}\right)} \left[(\mathbf{a}_Y - \mathbf{a}_X + \mathbf{d}_Y H) + \sqrt{(\mathbf{a}_Y - \mathbf{a}_X + \mathbf{d}_Y H)^2 + 4\mathbf{g}_X\mathbf{g}_Y} \right] \end{aligned} \quad (\text{A12})$$

The two steady-state growth rates are

$$\begin{aligned} \frac{\dot{GDP}}{GDP} &= \mathbf{a}_X + \mathbf{d}_X H + \mathbf{g}_X \frac{b_X}{b_Y} p_L^* \\ \frac{\dot{GDP}}{GDP} &= \mathbf{a}_X + \mathbf{g}_X \frac{b_X}{b_Y} p_H^* \end{aligned}$$

The difference between the right and left growth rate is

$$\mathbf{g}_X \frac{b_X}{b_Y} (p_H^* - p_L^*) - \mathbf{d}_X H$$

Extensions

The model as presented is designed to be as clean as possible and so omits many elements of the real economy. Some of the omissions represent matters of convenience, but a few are more fundamental. Three issues are clouded by this formulation. The first issue is that with a single input factor, the elasticity with which H is allocated across sectors depends only on substitution in preferences for output. It's not possible to look at factor substitution in production unless there is a factor to substitute. The more general model given here shows that a high elasticity of substitution in production also contributes to the existence of multiple growth states. The second issue is that the basic model ties large swings in factor allocation too directly to large swings in sector output. For example, in the near-Markov version described under 'A Limiting Model,' H is allocated entirely to one factor or the other. This implies, unreasonably, that only one of the two output goods is in production. With multiple input factors, swings in the factor determining growth need be only mildly related to swings in static production. The third issue clouded over is measurement of the relation between factor inputs and technical progress. With multiple input factors, swings in total factor input may be only mildly related to technical progress even though one particular input factor is closely related to technical progress.

To get these results we need more than one factor of production and we need the elasticity of factor demand for the non-growth-causing factor to be greater than the elasticity for H (so that swings in total factor input are smaller than swings in H). A simple way to arrange this is to assume that there are two additional factors, L^Y and L^X , each specialized for use in its respective sector. In this way, even though the additional factors are allocated in a competitive market, they are immobile across sectors in general equilibrium. I adopt a CES production function for each sector. The production function for Y is

$$Y = A_Y \left[b_{HY} H_Y^e + b_{LY} (L^Y)^e \right]^{\frac{1}{e}} \quad (\text{A13})$$

and analogously for X . The elasticity of substitution in production is $(e-1)^{-1}$. The model here reduces to the basic model when $b_{LY} = b_{LX} = 0$. Preferences are as given in equation (A2).

The condition that the ratio of prices equals the ratio of marginal costs is given by

$$p = \frac{MC_X}{MC_Y} = \left(\frac{A_Y}{A_X} \right)^e \left(\frac{Y}{X} \right)^{1-e} \left(\frac{b_{HY}}{b_{HX}} \right) \left(\frac{H_Y}{H_X} \right)^{e-1} \quad (A14)$$

For what follows, it is convenient to define Λ as

$$\Lambda \equiv \frac{b_{HY} H_Y^e + b_{LY} L_Y^e}{b_{HX} H_X^e + b_{LX} L_X^e} \quad (A15)$$

In the basic model, p depended only on the ratio of the technology coefficients. With multiple input factors, the expression for p is more complicated. It's easier here to use the ratio $A \equiv A_Y / A_X$ as the state variable. Equating the marginal rate of substitution to the marginal rate of transformation gives us

$$\left(\frac{A_Y}{A_X} \right)^r = \left(\frac{c_Y b_{HY}}{c_X b_{HX}} \right)^{-1} \left(\frac{H_Y}{H_X} \right)^{1-e} (\Lambda)^{\frac{e-r}{e}} \quad (A16)$$

which is analogous to equation (A4). To find $H_Y(A)'$, totally differentiate (A16). A convenient intermediate result is

$$\frac{1}{e} \frac{\partial \Lambda}{\partial H_Y} = \left[b_{HX} H_X^e + b_{LX} L_X^e \right]^{-1} \left[b_{HY} H_Y^{e-1} + \Lambda b_{HX} H_X^{e-1} \right] \quad (A17)$$

Totally differentiating (A16) gives

$$\frac{dH_Y}{dA} = \frac{r}{A} \left[(1-e) \left(\frac{H}{H_Y(H-H_Y)} \right) + (e-r) \Lambda^{-1} \frac{1}{e} \frac{\partial \Lambda}{\partial H_Y} \right]^{-1} \quad (A18)$$

As before, the condition for multiple equilibria is that there exist an equilibrium where $A(\mathbf{d}_Y + \mathbf{d}_X)H_Y' > A\mathbf{g}_X + \mathbf{g}_Y A^{-1}$. Again, it is easiest to look at the symmetric equilibrium (including the assumption $b_{HY} = b_{LY} \neq 0$). For the purpose at hand, assume that

$L^Y = L^X = H/2$. An equilibrium exists at $H_Y = H/2$. We have $\Lambda = 1$, $\frac{1}{e} \cdot \frac{\partial \Lambda}{\partial H_Y} = H_Y^{-1}$ and

$$\frac{dH_y}{dA} = \mathbf{r} \left[(1 - \mathbf{e}) \left(\frac{4}{H} \right) + (\mathbf{e} - \mathbf{r}) \frac{2}{H} \right]^{-1} = \mathbf{r} \frac{H}{2} [(1 - \mathbf{e}) + (1 - \mathbf{r})]^{-1} \quad (\text{A19})$$

The analogous point earlier shows that $H_y(A)'$ will be large – and multiple growth states will occur – if H is large or ρ is large. Equation (A19) shows that large ϵ , a high elasticity of substitution in production, also contributes.

A second problem of the single input factor assumption is that swings between growth states are tied too tightly to swings in sectoral output. We know that the latter are just not that large. Once we allow for multiple input factors, there is no difficulty in having large swings in the specialized factor which generates endogenous growth while seeing relatively small swings in other factors, so that sectoral output shifts are small. The limiting model here has $\mathbf{r} \rightarrow 1$ and $\mathbf{e} \rightarrow 1$. Examination of (A18) shows that dH_y/dA is infinite, so H is allocated completely to one sector or another. The H -less sector simply produces using its immobile factor. If the growth-inducing factor is relatively small in the economy, which is perfectly plausible, then swings between growth states will be accompanied by relatively small output swings between sectors.

Finally, turn to the question of measuring the relation between factor inputs and technical progress. For multiple growth states to exist, dH must be relatively large. There is at least some empirical evidence that learning-by-doing as a return to total labor input is modest. It is clear from equation (A13) that H may be a very small fraction of total input, measured as H plus L^Y weighted by the relative wage. Therefore, looking at movements of total factor input isn't particularly relevant. What does matter empirically is whether the growth due to 'engineering talent,' or whatever factor actually causes endogenous growth, is associated with sectoral swings in the allocation of that narrowly defined factor.